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THE PROBLEM OF SMALL MOTIONS OF A BODY WITH A CAVITY PARTIALLY FILLED WITH A VISCOUS FLUID

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The general problem of motion about a fixed point O_1 of a rigid body with a cavity partially or totally filled with a viscous incompressible fluid, under the action of gravity is studied here in its linearized approximation. Surface tension is neglected.

For the case of motion about the center of mass when the cavity is completely filled, this problem was considered in [1]. The general problem when the fluid viscosity is assumed to be small was considered in the paper of F.L. Chernous'ko [2].

1. Equations of motion of the fluid. We denote by Ω the region (in a moving coordinate system O_1xyz rigidly attached to the body) which is filled with the undisturbed fluid. We denote by Γ_0 the undisturbed free surface of the fluid, and by Γ_1 that part of the wall of the cavity in contact with the fluid. In the linearized approximation to the Navier-Stokes equations, the fluid motion is described in the O_1xyz system by

$$\frac{\partial \mathbf{u}}{\partial t} + \boldsymbol{\varepsilon} \times \mathbf{r} = -\nabla q + \nu \Delta \mathbf{u}$$
$$\left(q = \frac{p}{\rho} - \mathbf{g}\cdot\mathbf{r}\right) \qquad (1.1)$$

Here $\mathbf{u}$ is the relative velocity vector of the fluid particles, $\boldsymbol{\varepsilon}$ the body's angular acceleration vector, $\mathbf{r}$ the radius vector of the fluid particles relative to the point O_1, p the fluid pressure, ρ its density, $\mathbf{g}$ the acceleration due to gravity vector, and ν the kinematic viscosity.

Equation (1.1) is to be supplemented by:

the equation of continuity,

$$\operatorname{div} \mathbf{u} = 0 \qquad (1.2)$$

the condition of adherence of the liquid to the wall Γ_1

$$\mathbf{u}\,|_{\Gamma_1} = 0 \qquad (1.3)$$

the condition of absence of stress on the free surface Γ_2

$$\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} = 0, \qquad \frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y} = 0, \qquad \frac{\partial}{\partial t}\left(q - 2\nu \frac{\partial u_z}{\partial z}\right) = g u_z \qquad (1.4)$$

2. Equation of motion of the body. For the determination of the body's position we introduce a fixed coordinate system $O_1x_1y_1z_1$ and a system O_1xyz rigidly attached to the body, with common origin at the fixed point O_1. The two sets of axes may be brought into coincidence by means of three rotations (Fig. 1):

1) about $O_1\zeta$ by an angle δ_3, whereby the moving axes O_1x, O_1y go over to the semimoving axes $O_1\xi$, $O_1\eta$;

2) about O_1y_1 by an angle δ_2, whereby

$$O_1\xi \to O_1x_1, \quad O_1N \to O_1z_1;$$

3) about $O_1\zeta$ by an angle δ_1, whereby $O_1\zeta \to O_1z_1$, $O_1\eta \to O_1y_1$.

Thus the position of the body is determined by three independent angles δ_1, δ_2, δ_3. We call the vector $\boldsymbol{\delta}(t) = \{\delta_1, \delta_2, \delta_3\}$ with components in the O_1xyz system equal to the quantities δ_1, δ_2, δ_3, the vector of angular displacement.

In order to find the projection of the angular velocity vector $\boldsymbol{\omega}(t)$ of the body onto the axes O_1xyz, we first find its projection onto the semimoving axes $O_1\xi\eta\zeta$. With accuracy up to

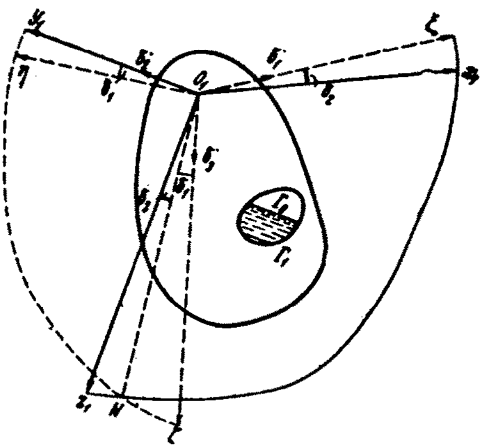

Fig. 1

terms of second order, they are $\omega_\xi = \delta^{\cdot}_1, \omega_\eta = \delta^{\cdot}_2, \omega_\zeta = \delta^{\cdot}_3$. In passing from the system $O_1\xi\eta\zeta$ to O_1xyz, they change in the following manner:

$$\omega_x = \delta^{\cdot}_1 \cos\delta_3 + \delta^{\cdot}_2 \sin\delta_3 = \delta^{\cdot}_1, \omega_y = -\delta^{\cdot}_1 \sin\delta_3 + \delta_2^{\cdot}\cos\delta_3 = \delta^{\cdot}_2,$$

$$\omega_z = \delta^{\cdot}_3.$$

Thus the angular velocity vector of the body in the system O_1xyz takes the form

$$\boldsymbol{\omega}(t) = \mathbf{i}\delta_1^{\cdot} + \mathbf{j}\delta_2^{\cdot} + \mathbf{k}\delta_3^{\cdot} \tag{2.1}$$

Here $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are unit vectors on the axes O_1x, O_1y, O_1z.

For the angular acceleration vector in this system, we obtain the following to within second order accuracy:

$$\boldsymbol{\varepsilon}(t) = \mathbf{i}\delta_1^{\cdot\cdot} + \mathbf{j}\delta_2^{\cdot\cdot} + \mathbf{k}\delta_3^{\cdot\cdot} \tag{2.2}$$

In the following we shall use (2.1) and (2.2) in the form

$$\boldsymbol{\delta}(t) = \int_0^{\tau} \boldsymbol{\omega}(t)\,dt + \boldsymbol{\delta}(0), \qquad \boldsymbol{\varepsilon}(t) = \frac{d\boldsymbol{\omega}(t)}{dt} \tag{2.3}$$

Suppose the only moment acting on the body is that due to gravity, and that is composed of the gravitational moment $\mathbf{M}_1$ calculated for the solid with the liquid imagined to be solidified, and the gravitational moment $\mathbf{M}_2$ arising as a consequence of the displacement of the fluid in the cavity. We compute the moment $\mathbf{M}_1$:

$$\mathbf{M}_1 = mg\mathbf{r}_c \times \mathbf{k}_1 \tag{2.4}$$

where $\mathbf{k}_1$ is the unit vector along the fixed axis O_1z_1, and $\mathbf{r}_c$ is the radius vector of the center of mass of the system relative to the point O_1. In the system O_1xyz it is equal to $\mathbf{r}_c = \{0,\ 0, a\}$ where a is the distance between the point of suspension and the center of mass.

By an immediate calculation we obtain

$$\mathbf{M}_1 = -mga(\mathbf{i}\delta_1 + \mathbf{j}\delta_2) \tag{2.5}$$

The moment $\mathbf{M}_2$ is (see e.g. [2])

$$\mathbf{M}_2 = -g\rho\mathbf{k}_1 \times \int_{\Gamma_0} \mathbf{r}f\,d\Gamma_0 \tag{2.6}$$

Here $z = f(t,\ x,\ y) + z_0$ is the equation of the disturbed free surface in the moving coordinate system (f is a quantity of first order of smallness).

The equation of motion of the solid takes the form

$$\frac{d\mathbf{L}}{dt} = \mathbf{M}_1 + \mathbf{M}_2 \qquad (\mathbf{L} = \mathbf{L}_1 + \mathbf{L}_2) \tag{2.7}$$

Here $\mathbf{L}_1$ is the angular momentum of the body with fluid solidified, and $\mathbf{L}_2$ is the relative angular momentum of the fluid.

With the aid of (2.5), (2.6), Eq. (2.7) may be written in expanded form as follows:

$$\mathbf{J}\cdot\boldsymbol{\varepsilon} + \rho\int_\Omega \mathbf{r}\times\frac{\partial\mathbf{u}}{\partial t}\,d\Omega + mga(\mathbf{i}\delta_1+\mathbf{j}\delta_2) + g\rho\mathbf{k}_1\times\int_{\Gamma_0}\mathbf{r}f\,d\Gamma_0 = 0 \qquad (2.8)$$

Here $\mathbf{J}$ is the moment of inertia tensor of the body-fluid system relative to the center O_1.

3. Formulation of the problem. We shall study the problem (1.1) -(1.4), (2.8) of determining the motion of the solid-fluid system under given initial conditions

$$\mathbf{u}(0, x, y, z) = \mathbf{u}_0(x, y, z),\ f(0, x, y) = f_0(x, y),\ \boldsymbol{\delta}(0) = 0,\ \boldsymbol{\omega}(0) = \boldsymbol{\omega}_0 \qquad (3.1)$$

For completeness we must add to the system a kinematic condition

$$u_z = df/dt, \qquad f(\tau, x, y) = \int_0^\tau u_z\,dt + f_0(x, y) \qquad (3.2)$$

For given $\mathbf{u}_0$ and f_0 on the free surface Γ_0 one determines q_0, and this, as was shown in [3] is sufficient for finding the initial values of those functions of which the desired velocity $\mathbf{u}$ is composed. In fact, from the boundary condition on Γ_0,

$$q + \mathbf{g}\cdot\mathbf{r} = 2\nu\frac{\partial u_z}{\partial z}, \quad \text{or} \quad q_0 = 2\nu\left(\frac{\partial u_z}{\partial z}\right)_0 + g(f_0 + z_0)$$

4. Reduction to an operator equation. To investigate the equations in the problem we introduce the function spaces treated in [3]. By $W_2^{1\circ}(\Omega)$ we denote the closure in the norm of the Sobolev space $W_2^1(\Omega)$ of the set of all solenoidal vector functions $\mathbf{v}$ in $W_2^1(\Omega)$ which vanish in a neighborhood of the surface Γ_1. By $L_2^\circ(\Omega)$ we understand the completion of $W_2^{1\circ}(\Omega)$ with respect to the norm of the space $L_2(\Omega)$. The orthogonal complement to $L_2^\circ(\Omega)$ in $L_2(\Omega)$ will be the closure of the set of all potential vector functions equal to zero on Γ_0 (see, e.g. [4]). We note that a vector function in $L_2^0(\Omega)$ has normal component zero on the boundary Γ_1.

The vector functions $\boldsymbol{\varepsilon}\times\mathbf{r}$ do not lie in the space $L_2^\circ(\Omega)$, and may thus be decomposed into two mutually orthogonal parts

$$\boldsymbol{\varepsilon}\times\mathbf{r} = \operatorname{grad}\varphi + \Pi(\boldsymbol{\varepsilon}\times\mathbf{r}), \qquad \Pi(\boldsymbol{\varepsilon}\times\mathbf{r}) \in L_2^\circ(\Omega)$$

From the above it follows that the function φ may be found as a solution of the following boundary value problem:

$$\Delta\varphi = 0, \quad \varphi = 0 \quad \text{on } \Gamma_0, \qquad \frac{\partial\varphi}{\partial n} = (\boldsymbol{\varepsilon}\times\mathbf{r}) \quad \text{on } \Gamma_1$$

Applying the method described in [3], one may reduce Eqs. (1.1), (1.2) with conditions (1.3), (1.4) to two operator equations in $L_2^\circ(\Omega)$:

$$\frac{d\mathbf{u}}{dt} + \nu A\mathbf{s} + \Pi(\boldsymbol{\varepsilon}\times\mathbf{r}) = 0 \qquad (4.1)$$

$$\nu \frac{d\mathbf{w}}{dt} + gT\Gamma\mathbf{u} = 0 \qquad (\mathbf{u} = \mathbf{s} + \mathbf{w}) \tag{4.2}$$

Here Γ is the operator associated with the trace of a vector-function on the free surface Γ_0, and A and T are operators generated by the auxiliary boundary problems described in [5]. The operator A will thus be a selfadjoint positive definite operator in $L_2^\circ(\Omega)$ with a completely continuous inverse.

Taking (2.3), (3.3) into account, we may write (2.8) in the form

$$\mathbf{s} + \rho \mathrm{J}^{-1}\int_\Omega \left[\mathbf{r} \times \frac{d\mathbf{u}}{dt}\right] d\Omega = - mga\mathrm{J}^{-1}\int_0^\tau (\mathbf{i}\omega_x + \mathbf{j}\omega_y)\,dt -$$

$$- g\rho \mathrm{J}^{-1}\int_0^\tau \left[\mathbf{k}_1 \times \int_{\Gamma_0} \mathbf{r}\Gamma\mathbf{u}\,d\Gamma_0\right] dt - g\rho \mathrm{J}^{-1}\left[\mathbf{k}_1 \times \int_{\Gamma_0} \mathbf{r} f_0\,d\Gamma_0\right] \tag{4.3}$$

Equations (4.1) - (4.3) constitute a complete system of operator differential equations for the three desired functions $\mathbf{s}(\mathbf{r}, t)$, $\mathbf{w}(\mathbf{r}, t)$ and $\boldsymbol{\omega}(t)$.

5. Integral equations. Existence theorems. We shall transform Eq. (4.1), the most complicated in the system (4.1) - (4.3). First, with the aid of (4.3) we eliminate $\mathbf{s}$ from (4.1). With accuracy up to terms of second order we obtain

$$(I + B)\frac{d\mathbf{u}}{dt} + \nu A\mathbf{s} = -\int_0^\tau F_1(\boldsymbol{\omega}, t)\,dt - \int_0^\tau F_2(\mathbf{r}, \Gamma u)\,dt - F_2(0) \tag{5.1}$$

$$\left(B\mathbf{v} = \Pi\left[\mathbf{r} \times \rho \mathrm{J}^{-1}\int_\Omega \mathbf{r}_1 \times \mathbf{v}\,d\Omega\right]\right)$$

$$F_1(\omega, t) = mga\,\Pi\left[\mathbf{r} \times \mathrm{J}^{-1}(\mathbf{i}\omega_x + \mathbf{j}\omega_y)\right]$$

$$F_2(r, \Gamma u) = g\rho\Pi\left[\mathbf{r} \times \mathrm{J}^{-1}\left[\mathbf{k}_1 \times \int_{\Gamma_0} \mathbf{r}\Gamma u\,d\Gamma_0\right]\right]$$

$$F_2(0) = g\rho\Pi\left[\mathbf{r} \times \mathrm{J}^{-1}\left[\mathbf{k}_1 \times \int_{\Gamma_0} \mathbf{r} f(0)\,d\Gamma_0\right]\right]$$

Here r_1 is the radius vector of the variable of integration, relative to O_1.

In [1] it is shown that the transport operator B will be a selfadjoint operator in $L_2^\circ(\Omega)$, and under the condition

$$J_1 < J_{33} \tag{5.2}$$

where J_1 is the polar method of inertia of the liquid relative to the fixed point O_1 and J_{33} is the least component of the moment of inertia tensor of the solid-fluid system relative to O_1, the norm of the operator B will be less than one.

Assuming (5.2), we apply to (5.1) the operator $(I + B)^{-1}$ and make further the following substitutions in all equations (5.1), (4.2) and (4.3):

$$\mathbf{u} = A^{-1/2}\xi, \quad \mathbf{s} = A^{-1/2}\eta, \qquad \mathbf{w} = A^{-1/2}\zeta$$

We note that the operator $\Gamma A^{-1/2}$ arising thereby is completely continuous [3]. Finally, substituting from Eq. (4.2) into the transformed Eq. (5.1) the expression for $d\zeta / dt$, we obtain

$$A^{-1/2}\frac{d\eta}{dt}+\nu(I+B)^{-1}A^{1/2}\eta=\frac{g}{\nu}T\Gamma A^{-1/2}(\eta+\zeta)-(I+B)^{-1}F_2(0)-$$
$$-(I+B)^{-1}\int_0^{\tau}F_1(\omega,t)\,dt-(I+B)^{-1}\int_0^{\tau}F_2(r,\Gamma A^{-1/2}\xi)\,dt \tag{5.3}$$

We now introduce into consideration the operator $A_0=A^{1/2}(\Gamma+B)^{-1}A^{1/2}$, which will be a selfadjoint positive definite operator on its natural domain of definition and will have a bounded inverse $A^{-1/2}(I+B)A^{-1/2}$. Suppose $e^{-\nu A_0 t}$ is the semigroup of bounded operators for which the operator νA_0 is a generator. We recall (see e.g. [6], p. 106), that the operator $A_0e^{-\nu A_0 t}$ will be bounded for $t>0$ and its norm satisfies the inequality

$$\| A_0e^{-\nu A_0 t}\|\leqslant c/t$$

From the above it follows that the operator $e^{-\nu A_0(t-\tau)}A^{1/2}$ for $t>\tau$ may be closed, and its closure is bounded. In fact, for $v\in D(A^{1/2})$

$$\| e^{-\nu A_0(t-\tau)}A^{1/2}v\|=\| e^{-\nu A_0(t-\tau)}A_0A^{-1/2}(I+B)v\|\leqslant\frac{c_2\|v\|}{t-\tau} \tag{5.4}$$

We apply the operator $\exp[-\nu A_0(t-\tau)]A^{1/2}$ to both sides of (5.3), replacing t by τ in that equation. Then on the left one obtains the derivative

$$\frac{d}{d\tau}(e^{-\nu A_0(t-\tau)}\eta)=e^{-\nu A_0(t-\tau)}\frac{d\eta}{dt}+\nu e^{-\nu A_0(t-\tau)}A_0\eta$$

We integrate both sides of this equation from 0 to $t-\varepsilon$. The resulting integral of the left side yields the expression

$$e^{-\nu A_0\varepsilon}\eta-e^{-\nu A_0 t}\eta_0$$

The integrand in the first term on the right will have the form

$$\frac{g}{\nu}e^{-\nu A_0(t-\tau)}Q(\eta+\zeta),\qquad Q=A^{1/2}T\Gamma A^{-1/2}$$

Here Q is a completely continuous operator in $L_2^{\circ}(\Omega)$ (see, e.g. [3]). The integral of this term from 0 to t will exist. The integrals of the remaining terms on the right will have the form

$$\int_0^{t-\varepsilon}\overline{\exp[-\nu A_0(t-\tau)]A^{1/2}}(I+B)^{-1}\int_0^{\tau}\Phi(\alpha)\,d\alpha\,d\tau$$

where $\Phi(\alpha)$ is a continuous function of α. This last expression may be transformed into the form

$$\int_0^{t-\varepsilon}A_0e^{-\nu A_0(t-\tau)}\int_0^{\tau}A^{-1/2}\Phi(\alpha)\,d\alpha\,d\tau=\int_0^{t-\varepsilon}\Big[\int_{\alpha}^{t-\xi}A_0e^{-\nu A_0(t-\tau)}d\tau\Big]A^{-1/2}\Phi(\alpha)\,d\tau\,d\alpha=$$

Cont. next page

$$= \frac{1}{\nu} \int_0^{t-\varepsilon} [e^{-\nu A_0 \varepsilon} - e^{\nu A_0 (t-\alpha)}] A^{-1/2} \Phi(\alpha)\, d\alpha$$

Finally, the last terms on the right, after being integrated, yield expressions of the form

$$\frac{1}{\nu} [e^{-\nu A_0 \varepsilon} - e^{-\nu A_0 t}] A^{-1/2} \Phi(0)$$

We thus arrive at the conclusion that after integration from 0 to $t - \varepsilon$ all terms on the left as well as on the right have limits as $\varepsilon \to 0$. Passing to the limit as $\varepsilon \to 0$ we finally bring the equation under consideration into the following form:

$$\eta = e^{-\nu A_0 t}\eta_0 - \frac{g}{\nu}\int_0^t e^{-\nu A_0 (t-\tau)} Q(\eta + \zeta)\, d\tau - \tag{5.5}$$

$$-\frac{1}{\nu}\int_0^t [I - e^{-\nu A_0 (t-\tau)}] A^{-1/2} [F_1(\omega, t) +$$

$$+ F_2(r, \Gamma A^{-1/2}(\eta + \zeta))]\, d\tau - \frac{1}{\nu}[I - e^{-\nu A_0 t}] A^{-1/2} F_2(0)$$

In Eqs. (4.2), (4.3) we perform the substitution $t = \tau$ and integrate with respect to τ from 0 to t. Thus

$$\zeta = \zeta_0 - \frac{g}{\nu}\int_0^t Q(\eta + \zeta)\, d\tau \tag{5.6}$$

$$\omega + B_1 A^{-1/2}(\eta + \zeta) = \omega_0 + B_1 A^{-1/2}(\eta_0 + \zeta_0) -$$

$$-\int_0^t (t - \tau)[F_3(\omega, \tau) + F_4(r, \Gamma A^{-1/2}(\eta + \zeta))]\, d\tau - tF_4(0) \tag{5.7}$$

Here

$$B_1 \mathbf{v} = \rho J^{-1} \int_\Omega [\mathbf{r} \times \mathbf{v}]\, d\Omega, \qquad F_3(\omega, \tau) = mgaJ^{-1}\{i\omega_x + j\omega_y\}$$

$$F_4(r, \Gamma A^{-1/2}\xi) = g\rho J^{-1}\left[\mathbf{k}_1 \times \int_{\Gamma_0} \mathbf{r}\Gamma A^{-1/2}\xi\, d\Gamma_0\right], \qquad F_4(0) = g\rho J^{-1}\left[\mathbf{k}_1 \times \int_0 \mathbf{r} f(0)\, d\Gamma_0\right]$$

Multiplying (5.5), (5.6) by the bounded operator $-B_1 A^{-1/2}$ and putting them together with (5.7), we reduce the system (5.6) - (5.7) to a system of equations of Volterra type with bounded operator kernels, for which the existence of a unique continuous solution is proved by the usual method of successive approximations.

The solution of the system (5.5) - (5.7) will consist of continuous functions with values in the spaces $L_2^\circ(\Omega)$ and R_3. The function constructed according to the formula

$$\mathbf{u} = A^{-1/2}(\eta + \zeta)$$

will have values in the space $W_2^{1\circ}(\Omega)$ for all t. Further differentiability properties of the functions $\mathbf{u}(t, \mathbf{r})$ and $\omega(t)$ were not studied; hence these functions should be considered as generalized solutions of the original problem.

6. The case of a completely filled cavity. In this case $\mathbf{u} = \mathbf{s}$, the equation (5.6) is dropped and the system (5.5) - (5.7) simplifies considerably; namely,

$$\eta = e^{-\nu A_0 t}\eta_0 + \frac{1}{\nu}\int_0^t [I - e^{-\nu A_0 (t-\tau)}] F_1(\omega, \tau)\, d\tau \qquad (6.1)$$

$$\omega + B_1 A^{-1/2}\eta = \omega_0 + B_1 A^{-1/2}\eta_0 - \int_0^t (t-\tau) F_3(\omega, \tau)\, d\tau \qquad (6.2)$$

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ON THERMOELASTIC STABILITY WITH SLIDING FRICTION

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Many works devoted to the investigation of the interdependence between the process of sliding friction and the normal displacement of a friction couple have appeared recently (the basic literature is presented in [1,2]). However, no questions connected with thermoelastic phenomena which can exert essential influence on the friction process in the presence of constraints limiting the normal displacement of the friction couple were considered in these works. The character of the thermoelastic processes occurring with friction is determined by the balance between heat liberation and heat elimination in the friction zone, and depends, in the long run, on the physical-geometric properties